

**UNIT I RANDOM VARIABLES**

**9+3**

Discrete and continuous random variables – Moments – Moment generating functions – Binomial, Poisson, Geometric, Uniform, Exponential, Gamma and Normal distributions.

**UNIT II TWO - DIMENSIONAL RANDOM VARIABLES**

**9+3**

Joint distributions – Marginal and conditional distributions – Covariance – Correlation and Linear regression – Transformation of random variables.

**UNIT III RANDOM PROCESSES**

**9+3**

Classification – Stationary process – Markov process - Poisson process – Discrete parameter Markov chain – Chapman Kolmogorov equations – Limiting distributions

**UNIT IV QUEUEING MODELS**

**9+3**

Markovian queues – Birth and Death processes – Single and multiple server queueing models – Little's formula - Queues with finite waiting rooms – Queues with impatient customers: Balking and reneging.

**UNIT V ADVANCED QUEUEING MODELS**

**9+3**

Finite source models - M/G/1 queue – Pollaczek - Khinchin formula - M/D/1 and M/E K/1 as special cases – Series queues – Open Jackson networks.

**TOTAL (L:45+T:15): 60 PERIODS**

**STAFF IN-CHARGE**

**HOD**

## COURSE PLAN

Sub.Code	:MA6453	Branch / Year/Sem:	B.E- CSE/II / IV
Sub.Name	: PROBABILITY QUEUEING THEORY	Batch	: 2016-2020
Staff Name	: Mr.K.BALAMURUGAN	Academic Year	: 2017-2018(Even)

### **COURSE OBJECTIVE:**

To provide the required mathematical support in real life problems and develop probabilistic models which can be used in several areas of science and engineering.

### **TEXT BOOKS:**

1. Ibe. O.C., "Fundamentals of Applied Probability and Random Processes", Elsevier, 1<sup>st</sup> Indian Reprint, 2007.
2. Gross. D. and Harris. C.M., "Fundamentals of Queueing Theory", Wiley Student edition, 2004.

### **REFERENCES:**

1. Robertazzi, "Computer Networks and Systems: Queueing Theory and performance evaluation", Springer, 3<sup>rd</sup> Edition, 2006.
2. Taha. H.A., "Operations Research", Pearson Education, Asia, 8<sup>th</sup> Edition, 2007.
3. Trivedi.K.S., "Probability and Statistics with Reliability, Queueing and Computer Science Applications", John Wiley and Sons, 2<sup>nd</sup> Edition, 2002.
4. Hwei Hsu, "Schaum's Outline of Theory and Problems of Probability, Random Variables and Random Processes", Tata McGraw Hill Edition, New Delhi, 2004.
5. Yates. R.D. and Goodman. D. J., "Probability and Stochastic Processes", Wiley India Pvt. Ltd., Bangalore, 2<sup>nd</sup> Edition, 2012.

### **WEB RESOURCES**

**W1.** <http://nptel.ac.in/>.

S. No	Topic Name	Books for Reference	Page No	Teaching Methodology	No. of Periods required	Cumulative no. of Periods
<b>UNIT - I RANDOM VARIABLES</b>						
<b>1</b>	Introduction to random variable	<b>T1</b>	59-61	BB	1	1
<b>2</b>	Pmf, probability density function	<b>T1</b>	62-75	BB	1	2

3	Moments	T1	85-101	BB	1	3
4	Problem based on MGF	T1	85-101	BB	1	4
5	Binomial distribution	T1	111-115	BB	1	5
6	Poisson distribution	T1	130-132	BB	1	6
7	Geometric distribution	T1	116-120	BB	1	7
8	Uniform distribution	T1	141-143	BB	1	8
9	Exponential distribution	T1	133-136	BB	1	9
10	Gamma distribution	T1	126-128	BB	1	10
11	Normal distribution	T1	144-147	BB	1	11
12	Summarizing the unit	T1	111-153	BB	1	12

### LEARNING OUTCOME

Application of one dimensional random variable in real life problem

### UNIT - II TWO-DIMENSIONAL RANDOM VARIABLE

13	Joint distribution	T1	167-173	BB	1	13
14	Marginal distribution	T1	167-173	BB	1	14
15	Conditional distribution	T1	178-182	BB	1	15
16	Problem based on marginal and conditional distribution	T1	173-175	BB	1	16
17	Covariance	T1	184-186	BB	1	17
18	Properties , problems on correlation	T1	184-186	BB	1	18
19	Properties , problems on correlation	T1	184-186	BB	1	19
20	Linear regression-properties	T1	418-422	BB		20
21	Problem on regression	T1	418-422	BB		21
22	Transformation of random variables	T1	197-215	BB	1	22
23	Problems on transformation of random variable	T1	-197-215	BB	1	23
24	Summarization of unit II	T1	167-190,197-215	BB	1	24

### LEARNING OUTCOME

Application of correlation and Regression in real life problem.

### UNIT - III RANDOM PROCESSES

25	Introduction, classification	T-1, ch-8	267-273	BB	1	25
26	Stationary process-wide sense stationary	TI	275-282	BB	1	26
27	Strict sense stationary	T1	275-282	BB	1	27

28	Markov process	<b>T1,R3</b>	358-359,309-316	BB	1	<b>28</b>
29	Markov chain	<b>T1</b>	359-376	BB	1	<b>29</b>
30	Markov chain	<b>T1</b>	359-376	BB	1	<b>30</b>
31	Problem based on markov process	<b>TI</b>	359-376	BB	1	<b>31</b>
32	<b>Transition probabilities</b>	<b>T1</b>	359-376	BB	1	<b>32</b>
33	<b>Transition probabilities</b>	<b>TI</b>	359-376	BB	1	<b>33</b>
34	<b>Poisson process-properties</b>	<b>TI</b>	342-356	BB	1	<b>34</b>
35	<b>Poisson process-problem</b>	<b>TI</b>	342-356	BB	1	<b>35</b>
36	<b>Summarization of unit</b>	<b>TI</b>	275-376	BB	1	<b>36</b>

**LEARNING OUTCOME**

Application of random processes in signal processing

**UNIT - IV QUEUING MODELS**

37	<b>Queueing system introduction</b>	<b>T2</b>	1-7	BB	1	<b>37</b>
38	Markovian models	<b>T2</b>	8-45	BB	1	<b>38</b>
39	Birth and death process	<b>T2</b>	1-45	BB	1	<b>39</b>
40	m/m/1 infinite capacity	<b>T2</b>	53-68	BB	1	<b>40</b>
41	<b>m/m/1 infinite capacity</b>	<b>T2</b>	53-68	BB	1	<b>41</b>
42	<b>m/m/1 finite capacity</b>	<b>T2</b>	53-68	BB	1	<b>42</b>
43	<b>m/m/1 finite capacity</b>	<b>T2</b>	53-68	BB	1	<b>43</b>
44	<b>m/m/c infinite capacity</b>	<b>T2</b>	69-73	BB	1	<b>44</b>
45	<b>m/m/c infinite capacity</b>	<b>T2</b>	69-73	BB	1	<b>45</b>
46	<b>m/m/c finite capacity</b>	<b>T2</b>	69-73	BB	1	<b>46</b>
47	<b>m./m/c finite capacity</b>	<b>T2</b>	69-73	BB	1	<b>47</b>
48	<b>Little formula, summarization of unit</b>	<b>T2</b>	53-115	BB	1	<b>48</b>

**LEARNING OUTCOME**

- Application of queuing models in real life problem

<b>UNIT - V ADVANCED QUEUEING MODELS</b>						
49	Finite source model	T2	209-260	BB	1	49
50	m/g/1 queue	T2, R3	209-260 336-343	BB	1	50
51	Pollaczek- Khinchin formula	T2	211-214	BB	1	51
52	Pollaczek- Khinchin formula	T2	211-214	BB	1	52
53	Problem on m/g/1 queue	T2	211-214	BB	1	53
54	m/d/1 and m/e/1 as a special cases	T2	209-260	BB	1	54
55	m/d/1 and m/e/1as a special cases	T2	209-260	BB	1	55
56	Series queues	T2	167-173	BB	1	56
57	Series queues	T2	167-173	BB	1	57
58	Open Jackson networks	T2,R3	174- 182,416- 422	BB	1	58
59	Open Jackson networks	T2,R3	174- 182,416- 422	BB	1	59
60	Summarization of unit	T2	167-260	BB	1	60
<b>LEARNING OUTCOME</b>						
<ul style="list-style-type: none"> <li>Application of queuing models in real life.</li> </ul>						

**COURSE OUTCOMES:**

**At the end of the course,**

1. The students will have a fundamental knowledge of the probability concepts.
2. Acquire skills in analyzing queueing models.
3. It also helps to understand and characterize phenomenon which evolve with respect to time in a probabilistic manner.

**CONTENT BEYOND THE SYLLABUS**

**INTERNAL ASSESSMENT DETAILS**

ASSESSMENT NUMBER	I	II	MODEL
Topic Nos.	1 – 24	25-48	1-60

Date			
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ASSIGNMENT DETAILS.

ASSIGNMENT	I	II	III
Topic Nos. For reference	1 – 24	25-48	49-60
Deadline	6.1.2018	8.2.2018	3.3.2018

ASSIGNMENT NUMBER	BATCH	DESCRIPTIVE QUESTIONS / TOPIC (Minimum 8 Pages)																				
I	B1 R.NoS.1-23	<p>1. Let <math>X</math> be a discrete random variable whose cumulative distribution function is</p> <p><math>F(X) = 0</math> for <math>x &lt; -3</math>  <math>1/6</math> for <math>-3 \leq X \leq 6</math>  <math>1/2</math> for <math>6 \leq X \leq 10</math>  <math>1</math> for <math>x \geq 10</math></p> <p>(a) Find <math>P(X \leq 4)</math>, <math>P(-5 &lt; X \leq 4)</math>,  <math>P(x = -3)</math>, <math>P(x = 4)</math></p> <p>(b) Find the prob. Mass function of <math>x</math></p> <p>2. A random variable <math>X</math> has the following probability function</p> <table style="margin-left: 40px;"> <tr> <td><math>X:</math></td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> </tr> <tr> <td><math>P(X):</math></td> <td><math>K</math></td> <td><math>3K</math></td> <td><math>5K</math></td> <td><math>7K</math></td> <td><math>9K</math></td> <td><math>11K</math></td> <td><math>13K</math></td> <td><math>15K</math></td> <td><math>17K</math></td> </tr> </table> <p>(i) Find the value of <math>K</math></p> <p>(ii) Find <math>P(X &lt; 3)</math>, <math>P(X \geq 3)</math>, <math>P(0 &lt; X &lt; 4)</math></p> <p>(iii) Find the distribution function of <math>X</math></p> <p>3. Moments of Binomial distribution about the origin.</p>	$X:$	0	1	2	3	4	5	6	7	8	$P(X):$	$K$	$3K$	$5K$	$7K$	$9K$	$11K$	$13K$	$15K$	$17K$
	$X:$	0	1	2	3	4	5	6	7	8												
$P(X):$	$K$	$3K$	$5K$	$7K$	$9K$	$11K$	$13K$	$15K$	$17K$													
B2 R.NoS.24-43	<p>1. Let <math>X</math> be a R. V with p.d.f given by</p> <p><math>f(x) = 2x</math>, <math>0 &lt; x &lt; 1</math>  <math>0</math>, elsewhere</p> <p>Find the pdf of <math>Y = (3X + 1)</math></p> <p>2. Find the moment generating function of an exponential random variable and hence find its mean and variance.</p> <p>3. Moments about the origin of Poisson distribution.</p>																					
II	B1 R.NoS.1-23	<p>1. Prove that a first order stationary random process has a constant mean</p> <p>2. Give an example of stationary random process and justify your claim.</p>																				
	B2 R.NoS.24-43	<p>1. Define a Markov chain (MC) . Explain: (i) How you would clarify the states and identify different classes of a MC. Give an example <math>t</math> each class.</p> <p>2. State the postulates of a poisson processes. State its properties and establish the additive property for the poisson process.</p>																				

III	B1 R.NoS.1-23	<p>1. Derive Pollaczek – Khintchine formula.</p> <p>2. Automatic car wash facility operates with only one Bay. Cars arrive according to a poisson process, with mean of 4 cars per hour and may wait in the facility’s parking lot if the bay is busy . If the service time for all cars is constant and equal to 10 min, determine <math>L_s</math>, <math>L_q</math>, <math>W_s</math> and <math>W_q</math></p>
	B2 R.NoS.24-43	<p>1. Customers arrive at a watch repair shop according to a Poisson process at a rate of one per every 10 minutes, and the service time is an exponential random variable with mean 8 minutes.</p> <p>i. Find the average number of customers <math>L_s</math> in the shop.</p> <p>ii. Find the average time a customer spends in the shop <math>W_s</math></p> <p>iii. Find the average number of customers in queue <math>L_q</math></p> <p>iv. What is the probability that the server is idle.</p> <p>2. A patient who goes to a single doctor clinic for a General check – up has to go through 4 phases. The doctor takes on the average 4 minutes for each phase of the check – up and the time taken for each phase is exponentially distributed. The arrivals of the poisson at the average rate of 3 per hour, what is the average time spent by a patient (i) in the examination? (ii) Waiting in the clinic?</p>

**PREPARED BY**

**LOGANATHAN.G**

**VERIFIED BY**

**HOD/CSE**

**APPROVED BY**

**PRINCIPAL**